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Infinite Humanity: A Counter to Saul Kripke's Skeptic

What is $117,428,931+328,853,209$? In all likelihood, you have never completed this computation before, and are therefore unable to appeal to some previous solution. Inputting into a calculator, you would reply with 446,282,140, but in virtue of what would this be the answer? You may reply that the calculator told you so, but in all likelihood you would have produced the same answer, after much labor, without aid of a calculator. You may simply reply that 446,282,140 is the sum, an instance of the rule you have applied many times previously. Here, Saul Kripke's skeptic steps in. Say you have never added numbers this large before. Another function, called quaddition and symbolized by '+*', returns $x+y$ for x and y less than 117,428,931 and returns 8 otherwise. How do you know that you have not been using the quaddition function throughout your life? Whenever you had previously written '2+3' you may have meant '2*3', both of which would return 5. Every other computation in your life would have given the same answer for addition and quaddition. In your past, the sum and the quum would be indifferntiable. If you simply follow the rule you have always followed to do the computation at hand, in virtue of what are you justified in replying 446,282,140 instead of 8?

Saul Kripke's *Wittgenstein on Rules and Private Language* opens with this puzzle. Kripke concludes that there are no facts about us in virtue of which we are justified to reply with the sum rather than the quum. Kripke then considers what he sees to be the strongest counter-argument against him, the dispositionalist account. According to the dispositionalist argument, to mean addition by '+' simply means that you are disposed to reply with the sum. So in this case, to mean addition by '+' in ' $117,428,931+328,853,209$ ' simply means that you are disposed to reply with 446,282,140, instead of 8. Those who agree with this argument are dispositionalists.

In the following discussion, I will first outline Kripke's three major counters to this dispositionalist account: our disposition to make mistakes, the finitude of our dispositions, and the descriptive quality of dispositions. I will show that these counter-arguments are not successfully combatted by expanding the discussion to humankind's dispositions. I will then present a dispositional argument, the argument from infinite humanity, that is immune to Kripke's criticisms. Is a member of an infinite humankind, which continually agrees on every computation, justified in claiming she means addition by '+'? I will argue that, unless Kripke gives a rigorous account of the normativity of mathematics that does not appeal to human action, Kripke will be forced to say yes.

Let me begin by outlining Kripke's objections to the dispositional account. Kripke's first criticism considers our disposition to make mistakes. Say I am terrible at computation, and always forget to carry the hundredths place. When I realize what I've done, I slap myself across the face and successfully give the sum, what I clearly was attempting to do all along. In that case, in virtue of what did I mean addition from the beginning? Dispositionalists, remember, would reply that I 'meant' addition by '+' if I am disposed to reply with the sum to any computation of the form ' $x+y$ '. However, this is clearly not the case, since my disposition sometimes leads me to forget to carry and thereby not give the sum. Therefore, dispositionalists would be forced to accept that I had actually 'meant' another function, which pointwise matches my dispositions. So, according to the dispositionalist, I did not mean addition by '+' afterall.

How would a dispositionalist define "mistake"? A mistake is the disposition to give an answer different from that which matches the function I 'meant'. But this explanation plunges us into a vicious circle. Admitting that I made a mistake forces me to rigorously justify that I actually 'meant' addition, which falls victim to Kripke's original skeptical argument.

Even more fundamental than Kripke's first criticism, we do not have dispositions for all 'x+y' expressions. You likely replied to '117,428,931+328,853,209' with a shrug, or simply continued reading because you did not have an immediate idea. Some numbers are simply too large for us to have a disposition. Some are even too long for us to hear in a lifetime, and therefore no matter how practiced in mathematics we are, we would be unable to have any disposition. Likely there is a certain point, call it w , after which numbers are too large for us to have a disposition. Define a new function called taddition which, for all x and y less than w , returns 'x+y' and returns 6 otherwise. In virtue of what have you 'meant' addition instead of taddition in the past?

Such an objection may seem to highlight a limit to human cognition, and therefore may appear to be more of a comment on our 'hardware', rather than what we 'mean'. Imagine, then, that humans were given whatever means needed to compute any computation provided to them. The dispositionalist would reply that if I were given the means, I would provide the sum. But as Kripke reveals, this "presupposes a prior notion of my having an intention to mean one function rather than another by '+'" (28). It is true that if I meant addition by '+' I would, given the means, reply with the sum. But it is similarly true that if I meant quaddition by '+' I would provide the quum. Asserting the first of these conditionals instead of the second just forces us, yet again, to ask in virtue of what did I mean addition by '+' initially?

Even if the dispositionalist were to successfully retort Kripke's error and finitude counter-arguments, Kripke sees something inherently flawed in the dispositionalist account of what we 'mean'. A disposition to give a certain response can be seen as an assertion that, given the chance, this is the response one would give. But is this the response one *should* give? Ascribing a rule to the computation, whatever rule it may be, is normative. That rule dictates what the result should be, not simply the response we are disposed to provide. In this

way, the dispositionalist account does not address the actual justificatory task at hand. The skeptic asked you to provide a justification for $117,428,931+328,853,209$ being $446,282,140$, and saying “that’s the answer I would give if asked” does not justify anything. According to Kripke, the dispositionalist account is purely descriptive, and therefore unable to sufficiently explain the normative, justificatory challenge of the skeptic.

What type of justification would actually be appropriate, if one were to exist? While my individual disposition to provide the sum rather than the quum does not justify anything, there seems to be a universality to that tendency. I surmise that it would be hugely difficult to find anyone who would reply with 8 to $117,428,931+328,853,209$. Likely, everyone would, after much effort, reply with the sum. Does this give any justification to the claim that I *should* give the sum, rather than the quum?

It seems entirely natural to argue that popular opinion may contribute justification. We could easily imagine a world where we all reply with 8, and most of us would agree that in that world we could mean something different by ‘+’ than we would in the world where we reply with $446,282,140$. One could argue that, in that world, our dispositions show that we mean quaddition by ‘+’. Our communal practices may determine our meaning, since different communal practices seem to result in different meanings. Therefore, community-wide dispositions may provide justification for our meaning addition by ‘+’ rather than quaddition.

Clearly, though, the skeptic could reply with a larger skeptical claim about huge mistake-making. The skeptic could argue that, since individuals frequently make computational mistakes, humankind as a whole may be making a large-scale error by replying with the sum, when in reality we intend to use the quum. At some point in the future, we may all realize our error, slap ourselves across the faces, and finally give the quum, the function we intended all

along. But, like with the individual case, this error argument would be just as susceptible to the original skeptical argument. In virtue of what can we argue that we intend to use the quum?

While Kripke cannot easily argue that all of humankind is making a mistake, his finitude objection persists. Say humans agreed on every computation ever completed. Imagine that humans always agreed on the solution to problems involving '+', no matter the situation. In this case, we are stipulating away all instances of 'error.' As we all know, humankind is finite. There have been a finite number of humans on earth, each of whom only lived for a finite number of years. Therefore, only a finite number of computations have been performed in human history. Thus there exists a least number, X, such that no humans have computed with numbers larger than X. Then imagine the function, maddition, which returns the sum for numbers less than or equal to X, and returns 5 for numbers greater than X. Following the skeptic's original argument, no fact about the history of humankind justifies the claim that, by '+', we mean addition rather than maddition.

While humans, when presented with '117,428,931+328,853,209', are more likely to reply 446,282,140 rather than 8, this is consistent with humans as a whole using the function maddition instead of addition. Therefore, despite community-wide agreement seeming like justification for the claim that by '+' I mean addition, humankind as a whole is just as susceptible to the original skeptical argument. Consequently, following Kripke's original discussion, once the argument is brought to the scale of all of humanity, there are no facts about our dispositions which provide the required justification.

But would these objections to the humanity-wide dispositional argument hold if humankind were infinite? Imagine an infinite humankind. A humankind that has existed for an infinite amount of time, and whose members live infinite lifetimes. Imagine, also, that these humans have computed and remember every computation with '+' possible, and that they agree

on every result, providing the sum without deviation. Would a human in this world be justified in claiming she means addition by '+'?

Kripke's error counter-argument fails as much here as it does when considering finite humanity; it is impossible to claim that this infinite humanity is making a community-wide error without entering a vicious circle. Similarly, in their infinite lives, the humans work tirelessly on their computations, ultimately agreeing universally across humanity. Therefore, Kripke's counter-argument about individual error cannot be applied here.

The finitude of an individual's dispositions is no longer problematic either, since a person would have the time and capacity to perform any computation presented to her. Similarly, humankind would have, at some point, performed every computation possible. Therefore there is no maddition-style function that is defined in terms of the finitude of humanity's calculations. These humans have dispositions about every computation possible, and therefore there is no finitude problem.

The only counterargument that may continue to hold seems to be the discussion of the necessarily descriptive nature of dispositional arguments. Is an individual in this world able to justify the claim that she means addition by '+' by appealing to the infinite communal disposition to provide the sum? The appeal of the previous argument from humanity-wide agreement seems to become even stronger now that humanity is infinite. Humankind continually agreeing, forever and without deviation, to provide the sum when presented with '+' seems like justification that an individual human should provide the sum too. If these humans are unjustified in claiming they mean addition by '+', normativity must not be explained by human behavior. For Kripke to successfully argue that these infinite humans are unjustified in saying they mean addition by '+', he would need to give an account of the normativity of mathematical laws which

does not cite human actions, a discussion that is wildly beyond the scope of Kripke's original argument.

By creating a series of functions that are indistinguishable from addition in all cases other than those we have not yet computed, Kripke shows us that there is nothing in our history to justify that we 'mean' addition by '+', rather than one of these alternate functions. While it seems natural to appeal to our "disposition" to give the sum when trying to find such a justification, Kripke identifies three errors in this explanation: the mistakes we are disposed to make, the finitude of our dispositions, and the necessarily descriptive quality of dispositional accounts. In response to this, I presented a counter-argument from humanity-wide agreement. However, I showed this counterclaim to be just as susceptible to Kripke's three criticisms. Not only is there nothing in my personal history to justify the claim that I mean addition by '+' rather than quaddition, there is also no fact about the history of humankind which shows that we are not all using an alternate function. I finally presented a hypothetical infinite humanity whose communal actions may provide the desired justification for meaning addition by '+'. Ultimately I concluded that Kripke would need to give a rigorous account of the normativity of mathematics to be able to ward off such a counterargument, a task that was understandably beyond the scope of his original work.