May 20, 2017

The Peirce Papers: Problems with Alpha Graphs

How did your first logic professor disambiguate between sentential and quantificational logic? Likely, she explained that sentential calculus formalizes reasoning about full sentences, while quantificational logic delves into the internal structure of sentences themselves. These classifications intrinsically bind logic and language, the former being defined in terms of the latter. Systems of logic are made up of their formal language, the set of symbols within the system. Logical properties, such as validity, are taken to be properties of the sentences of these formal systems. In this way logical properties are defined by their relationship to a language. But is logic best understood as a property of language, of sentences and their internal structures and relationships? Or instead, should logic be seen as a property of the world independent of language, that is exemplified by sentences of a language?

Our classical logical notation perpetuates this binding of logic to language. This binding distorts our perception of logic, as linguistic conventions veil logical properties. Since current logical notation is written, it is constrained by a certain directionality. One typically works from left to right, forcing statements such at A&B to be necessarily different from B&A, despite their natural logical equivalence. Rules of inference are required to move from one to the other (Dilbert). Similarly, quantification is limited by writing a certain variable over which to quantify. Therefore, despite **for all x, Fx** and **for all y, Fy** clearly being logically equivalent, typical symbolic systems require transformation rules here as well (Dilbert). These two examples exhibit the divergence between logic and language. If A&B and B&A are logically equivalent, then that logical property of equivalence exists independent of language, since it is the language that makes the two schemata distinct.

This shows language, here the use of letters to abbreviate and instantiate, actually retreats us from logic. Logical equivalence seems to exists independent of the language with which schemata are expressed, and therefore the limitations posed by linguistic expression actually shroud logical properties. The difference in the notation between A&B and B&A veils their logical equivalence. These examples hint that our understanding of the subject of logic itself is distorted by our focus on language.

Charles Sanders Peirce, one of America's most prolific logicians, presented a graphical representation system he believed displayed logical properties more overtly, instead of blurring them by language. He sought a more natural system with which to formalize reasoning, as oppose to facilitate it. Tracking the development of his new systems is historically difficult, however, as the majority of his work rests, handwritten and disassembled, in Harvard University's Houghton Library. The primary thread on which to begin remains the notes for his Lowell Lectures of 1903, where he introduced his three-pronged diagrammatic method.[1] Peirce's Existential Graphs (EG) began with a propositional logic foundation, upon which quantificational and modal logics were built. This foundation, called the Alpha Graphs, will be my primary focus. I will lay out the

[1] Peirce openly referenced one of his predecessors as inspiration for this aim, striving for the formalization of Euclid's *Elements*. Famously, though, despite the similarity of their goals, there is no evidence that Peirce read Gottlob Frege's formalized language.

signs, rules, and decision procedure of the Alpha Graph system, and then consider three objections to the rigor and strength of the system.

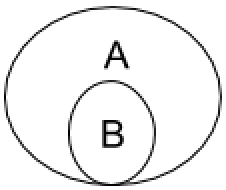
Ultimately, I will argue that Peirce does make strides toward freeing logic of its traditional linguistic chains. However, his system does not complete the journey. His graphs present complexity that makes his system more cumbersome than necessary. More importantly, his graphs still involve the abbreviation of sentences into sentence letters, and therefore continue to tie logic to language. In this way, Peirce's system does not free logic completely, as he would have liked. Therefore, while Peirce's goals were important and admirable, modifications to his system would need to be presented to rid logic of its linguistic chains. I will end by discussing the features required of a system for that system to genuinely free logic from language.

Let me begin by outlining the general system of Alpha Graphs. Let the blackboard represent the "sheet of assertion" (SA), meaning the universe of discourse.[2] As one can both write and draw upon the SA, we will refer to any act of asserting as "scribing" (Lecture 2 Volume 1). Whenever anyone scribes upon the SA, it makes the universe more determinate. This is due to any scribing upon it being an assertion of "existence," which explains the term "Existential Graphs."

Peirce gives two signs which depend on the existence of the SA, the "scroll" and "putting two graph replicas into the same area" (Lecture 2 Volume 2).[3] Let me explain

[2] As these were presented in his lecture notes, he specifically references the "blackboard," but this should be understood to be any surface upon which one is "scribing" (Lecture 2).
[3] By graph replica, Peirce means to make a distinction between a specific writing of a graph and the graph itself. This type/token distinction is similar to the word "the" and

each of these in turn. The scroll represents the "conditional de inesse," the material conditional. It is assembled by two concentric "cuts," in the SA. The statement written in the outer section of the scroll is the antecedent of the conditional, and that written in the inner section is the consequent.



scroll: If A, then B.

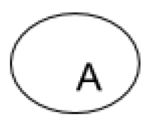
Importantly, this is not like a Venn Diagram, with the assertions in the center somehow being a subset of those in the outer section of the scroll. The conditional should not be seen as a diagram of a range of possibilities but instead an actual "state of things… if A is true, then B is true" (Lecture 2 Volume 1). As this idea of a singular state of the world was of utmost importance to Peirce, he believed his scroll to be the "one proper way of expressing" the material conditional diagrammatically (Lecture 2 Volume 1). So, scrolls should be read as "if A then B".

By "putting two graph replicas into the same area," Peirce means that scribing two graphs on the SA asserts them both simultaneously. So the following graph represents "A and B": A B . This visual juxtaposition representing conjunction embodies the natural reasoning Peirce intended in his diagrammatic method, and therefore is an obvious benefit of the Alpha Graph system.

the typed word "the" written on this page, the first being analogous to the graph and the second to the graph replica.

As Peirce understands his diagrams to be topological objects, with topology being “the study of the continuous connections ...which are free to be distorted in any way so along as the integrity of the connections ... is maintained” (Peirce 4.219 from Kent), when one scribes two assertions it does not matter where on the SA they are written, as long as their “integrity” is preserved. In Peirce’s system, any two graphs are equivalent if and only if they are topologically equivalent. Therefore, moving a graph on the SA does not change it. As scribing two graphs on the SA asserts them both simultaneously regardless of their placement, the problematic A&B, B&A distinction in traditional logic is avoided.

Peirce defines the scroll itself explicitly, and then defines negation in terms of the scroll. Each cut of the scroll can be seen as negating what is asserted inside. Peirce then introduces cuts on their own representing a singular instance of negation. For example, “A pear is ripe,” surrounded by a cut, represents “it is false that a pear is ripe.”

So, “~A” should be represented as: 

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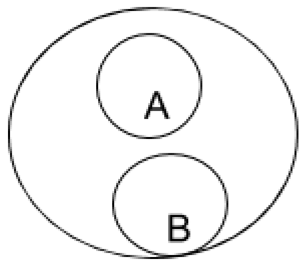
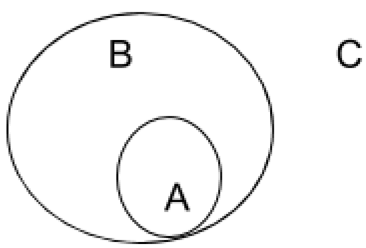
Putting these two notions together, the scroll becomes a negation of the two assertions within the outer cut, the antecedent and the negation of the consequent, since the consequent is surrounded by the inner cut. This mirrors the material conditional, “if a then b,” being “not (a and not b)”. As juxtaposition and cuts represent conjunction and negation, Peirce’s system is clearly expressively adequate.

Let me present a few examples of sentential calculus sentences and their Alpha Graph counterparts. To begin, the disjunction is relatively cumbersome within Peirce's system. "a or b" needs to be represented as:



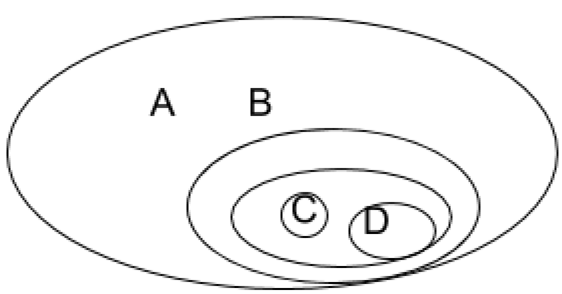
"if ~a, then b"

This is the traditional "if a then b" scroll, but with "a" surrounded by a cut, leading to "if ~a, then b". While juxtaposition within Peirce's system naturally represents conjunction, the disjunction is not so easily represented. To see the combination of disjunction and conjunction, consider "(a or ~b) and c":



"(if b then a) and c"

Finally, to exhibit conjunction and disjunction within a conditional, consider "if (a and b) then (c or d)". In normal notation, this is quite simple looking. However, in Peirce's system, the representation gets complicated quickly:

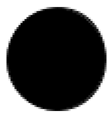


"if (a and b) then (if ~c then d)"

This complexity arises from Peirce's system not using any additional symbol for disjunction. Within the classic system, disjunction is clearly an unnecessary addition, since the conjunction, negation and conditional already provide the necessary expressive adequacy. But, despite the disjunction being yet another symbol added to the system, it removes complexity within logical statements, as can be clearly seen in the difference between "if (a and b) then (c or d)" and "if (a and b) then (if ~c then d)". Peirce's system involves no such shorthand symbol for disjunction, and in this way, while there are fewer symbols within the system, complexity is added to individual schemata.

In his explanation of the scroll being two instances of negation, Peirce appeals to topology. Cuts are supposed to be seen as actually cutting through the SA, so that "the whole of its interior is severed from that sheet" (Lecture 2 Volume 1). Imagine the interior of the scroll's outer cut being on a separate level, above the SA. The inner section of the scroll, however, should be seen as a "patch," bringing it back onto the SA. See it as a doughnut sitting on a table. The table is the sheet of assertion, the doughnut is the negated, outer section, and the doughnut hole is the inner section back on the table. So for an assertion to be made in the inner section, it is on the table, surrounded by the raised outer section. The outer section is disjoint from the SA, while the inner section remains on the SA. This mirrors the topological notion that when one passes through a boundary, one moves from the exterior to the interior, and then by passing through boundary again one passes back to the exterior (Dilbert). This inspired Peirce's use of double negation elimination, which will be introduced soon.

The final object in Peirce's system presents a point of confusion. According to Peirce, the "blot" or "pseudograph" is a fully saturated area which represents "all statements being true" in the blotted area (Lecture 2 Volume 2).

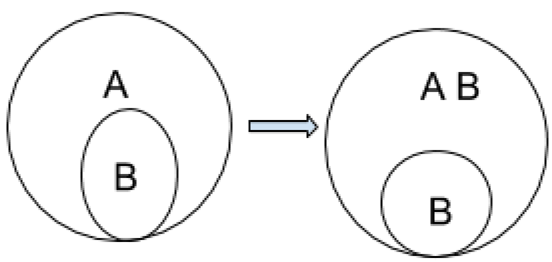

blot

While he uses the blot as if it were a tautology throughout his work, the blot itself does not fit easily within his graphical system. The blot should be seen as a fully saturated, drawn in section of the SA. Since it is fully saturated there, nothing else can be asserted on the SA within that area. This means that everything has already been asserted and therefore that everything is true. While Peirce intended the blot to represent a tautology, something which is always and necessarily true, "all statements being true" more naturally signifies a contradiction. A contradiction implies everything, therefore all statements are true once a contradiction is asserted. This directly contradicts Peirce's personal use of the blot in proofs.

This conceptual contradiction need not act as a fundamental problem with Peirce's system, as a simple solution is readily available. As I have already explained, the oval represents negation. Therefore, imagine a fully saturated area representing a contradiction, the most natural representation of such an object. Then surround that saturated area with an oval. This visually would make no difference, a blot with an oval boundary and a blot without one would both look the same. However, with this clarification, the closed blot would represent the negation of a contradiction, namely a tautology. This allows Peirce's system to remain visually unchanged, while preventing the confusion about the blot's meaning. Despite this opening the door to the possibility of both open and closed blots, I would limit the system simply to the closed blot, representing the tautology, since the two types of blots would be indistinguishable visually. For ease of comprehension in the derivations soon to be presented, read the blot as being a closed blot, representing a tautology.

Peirce then presents the "alpha permissible transformations" of graphs, so named because they cannot change "a true graph into a false one" (Lecture 2 Volume 1). He claims that these rules, which are essentially rules of inference, can somehow be "proved by the principles of the alpha part of the system to be permissible" (Lecture 2 Volume 2). This statement could be interpreted as making a Gödelian claim of proving metamathematical claims about the system using the system itself, but this is neither Peirce's intention nor historically coherent. The only "principles of the alpha part of the system" that are not the transformation rules are the signs, which, being static objects, cannot prove anything. I believe Peirce to be making a semantic claim of the soundness of his rules, without explicitly invoking an anachronistic formal notion of soundness. He seems to be saying that, using the meanings of the system's signs presented above, the transformation rules do not change the truth of a graph. This is not proving the permissibility of the rules "by the principles of the alpha part of the system," but instead showing their permissibility using the semantic meaning of the signs. While the idea of proving metamathematical claims about a system using the system itself plays a large historical role in the period to follow, I do not believe Peirce to be truly making this type of assertion.[4]

Peirce presented his transformation rules as three paired rules. The first double rule, the Rule of Erasure and Insertion, goes as follows: “Within even cuts (or none) any graph can be erased; and within odd cuts inserted” (Lecture 2 Volume 2). In terms of terminology, “even” and “odd” cuts signify an even number and an odd number of cuts surrounding a graph respectively. The first section of this rule allows one to assert two conjuncts separately when presented with a conjunction, with conjunction in the system being the juxtaposition of graphs. To understand the second part, let me appeal to an example:



In a scroll, one can repeat the inner proposition in the outer section, turning “if a then b” into “if a and b then b.” This clearly does not change the truth of the graph.

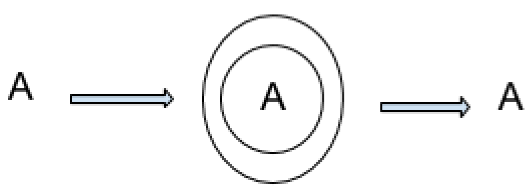
The second double rule, the Rule of Iteration and Deiteration, allows us to iterate, onto the SA or within any cuts already present, any graph already scribed, and “two replicas of the same graph one of which is enclosed by every cut that encloses the other, the former may be erased” (Lecture 2 Volume 2). The second section of this rule

[4] It is possible that Peirce was referencing the decision procedure to come, which can be proven using the transformation rules of the system, but in context this seems unlikely.

permits erasure of one graph replica identical to another graph replica on the SA, as long as they are surrounded by the same number of cuts. Both parts of this rule clearly preserve truth.[5]

The final double rule, the Rule of Insignificants and the Pseudograph, is essentially a rule of double negation and tautologies. The rule says, “a double enclosure can be circumposed about any graph or be removed from any graph; and any enclosure containing a vacant cut or other form of pseudograph [a blot] can be suppressed or inserted” (Lecture 2 Volume 2). This essentially says that double negations and tautologies can be added or removed anywhere, which clearly cannot make a true graph false. To be clear, we cannot add two cuts wherever we would like. The double negation section of the rule allows us to surround any graph by two concentric ovals, with nothing in between them, as can be seen below.

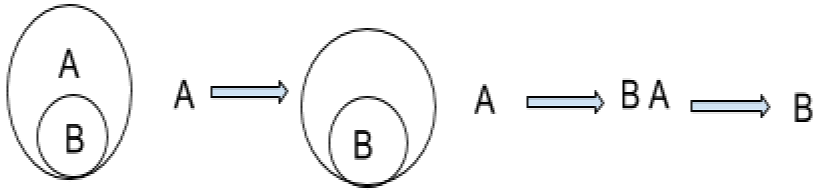
Examples of Permissible Uses of the Rule of Insignifacants and the Pseudograph:



These “transformation rules,” or rules of inference, are clearly sound, as they preserve truth. This can be shown rigorously, but I do not see that to be necessary here (Roberts 1992). See the below diagram for an example of each of these rules in action, in the case of modus ponens:

[5] It is important to note here that a false graph can be transformed into a true one using these rules. However, Peirce did not define permissible transformation in terms of preserving truth value as much as preserving truth. Also, as Peirce does not see the scroll as representing a disjunction of possibilities, but instead as a single “state of the world,” he likely would not see the consideration of graphs that begin as false to be important.

Premises *By deiteration* *By double negation* *By erasure*



At the end of his second lecture, Peirce outlined an explicit decision procedure for alpha possibility so simple that it would be “easy to define a machine that would perform it” (Peirce from Roberts 1997).[6] Peirce defines three categories within which all graphs fall: alpha impossible, alpha necessary and alpha contingent (Roberts 1997). These mirror the notions of logical contradictions, tautologies, and contingencies within Peirce’s system. To be clear, though, these are specific to his alpha system, and therefore only address propositional logic.[7]

Peirce then defines a clear procedure for determining into which of these categories any graph falls. The procedure involves separating the graph in question into two sections, and performing a set of operations derived from the transformation rules until explicit cases of truth value are present in the upper section. The operations go as follows (Roberts 1997):

1) Cancel any empty enclosures.
2) Cancel any double cuts.
3) Transfer one otherwise unenclosed letter in the lower region to the upper region, and cancel it wherever else it occurs in the lower region.
4) For each letter that is the sole occupier of a cut in the lower region, cancel each of its instances in the lower region and replace them all by an empty cut. Then insert in the upper region that letter surrounded by one cut.

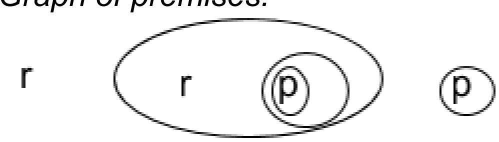
[6] As I could not find the notes on this procedure, I am grateful for Roberts 1997 outline. Peirce’s third lecture notes explicitly say, “I explained at the end of the last lecture how to ascertain speedily and certainly whether a graph is absurd” (Lecture 3 Volume 1). However, pouring through all of his more than 100 page Lecture 2 notes, I cannot find anything on this procedure. Roberts’s citation method does not match any manuscript in the collection, so I am going to appeal to his explanation exclusively.
[7] He does not give decision procedures for his beta and gamma graphs.

5) When operations 1-4 are performed until they can no longer be performed, if there are any graphs remaining in the lower region they will consist exclusively of letters enclosed in different numbers of cuts. For each of these letters, "(a) iterate on the once-enclosed area (alongside … the antecedent) every enclosure which occurs there… (b) In the upper region of each of the original enclosures scribe the letter… cancelling… throughout the lower region… (c) In the upper region of each of the copies scribe an enclosure containing only the letter… and throughout the lower region substitute an empty cut for that letter" (Roberts 1997).

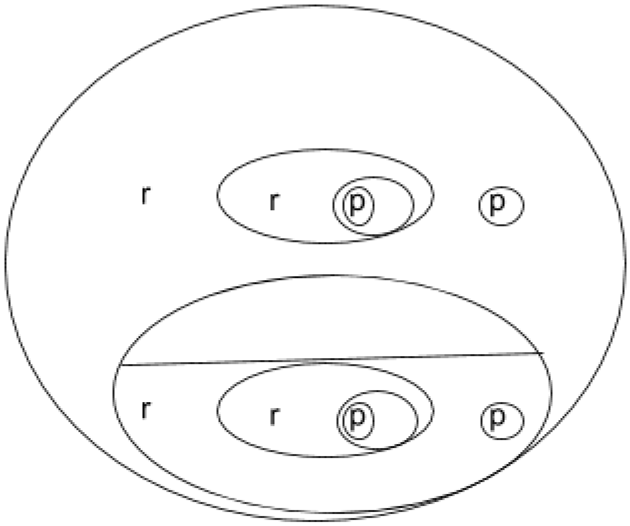
When these operations have been performed exhaustively, no graphs will be present in the lower region. If the original graph is contingent, each proposition or its negation will appear in the upper region, signaling the truth values required for the graph to be true, as this "is understood as referring to the universe of alpha possibility" (Peirce from Roberts 1997). See the diagrams below for the decision procedure in action (Roberts 1997):

Premises:
"It rains." → "r"
"If it rains, it does not pour." → "if r, then not p"
"It does not pour." → "not p"

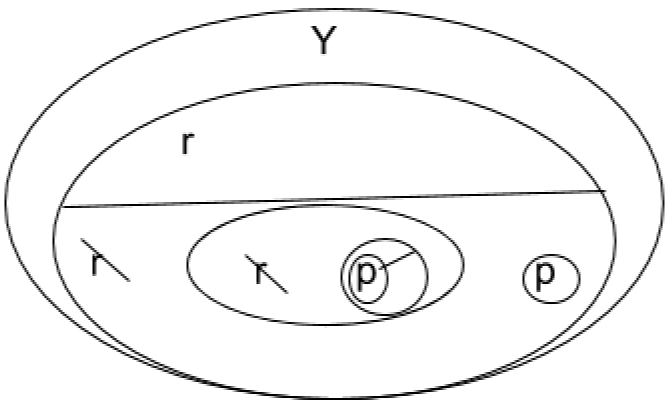
Graph of premises:



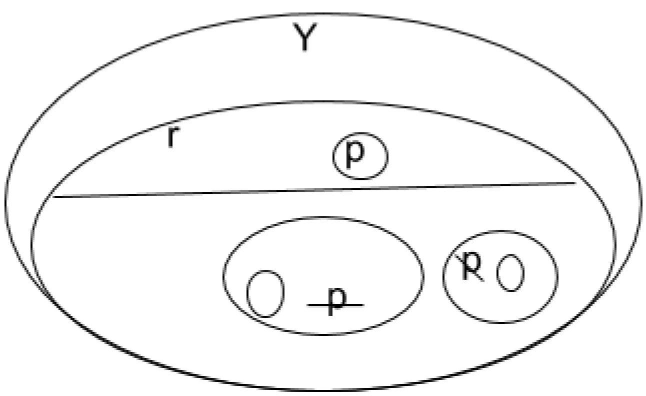
Step 1: Construct a graph of "if Y then Y" with Y being the conjunction of premises. Divide the consequent into two sections. This dividing line is simply for working purposes, and is not a sign of the system.



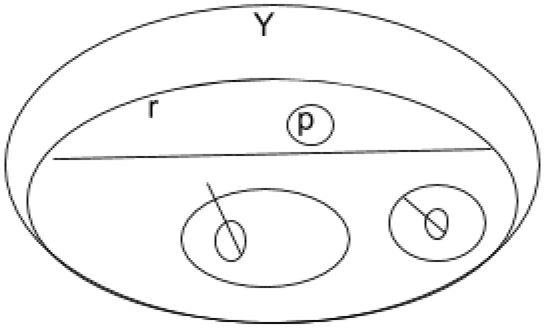
Step 2: For simplicity, we can replace the antecedent with Y from now on. Cancel the double cut around "p" by (2). Transfer the otherwise unenclosed "r" in the lower region to the upper region and cancel its other occurrences in the lower region by (3). For pedagogical purposes, I will cancel using a line, but in reality I would erase.



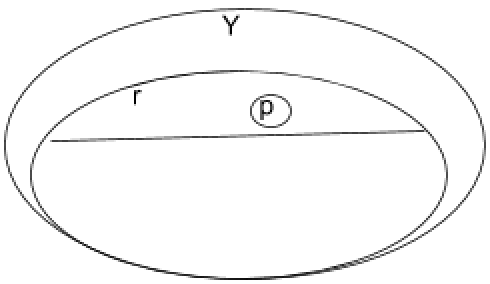
Step 3: This results in two occurrences of "p" surrounded by one cut in the lower region. Transfer "p" surrounded by one cut to the upper region and replace each instance of "p" by empty cuts in the lower region by (4).



Step 4: Cancel the two double cuts, by (2).



This results in the following final diagram, which can be read as a normal conditional:



If Y, then r and not p.

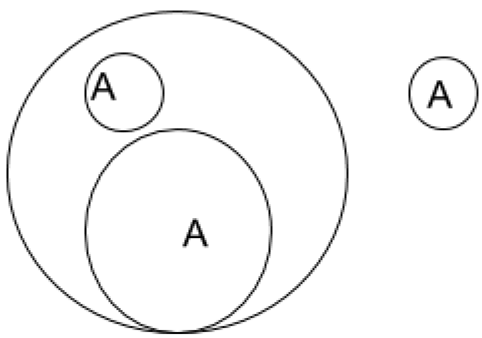
Therefore, the truth values of the sentence letters to make the conjunction of premises (Y) hold are "r" being true and "p" being false. "It rains" is true and "it pours" is false.

If the graph is impossible, then there will be neither letters nor letters within cuts in the upper region, signaling that no truth assignment of the propositions will make the graph in question true. As Roberts notes in "A Decision Method for Existential Graphs," this procedure narrows relevant cases in a method similar to W.V. Quine's Truth Value Analysis from *Methods of Logic,* until only propositions and negations of propositions remain (Roberts 1997).[8]

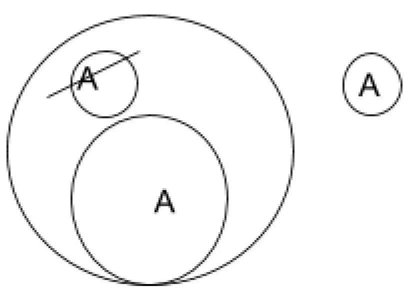
This seems like an overly cumbersome way to identify simple logical structures like contradictions and tautologies, however. As with the classical logical system, one can frequently identify contradictions simply from the logical structure itself, and by

[8] In my research I actually stumbled upon W.V. Quine's personal copy of "The Origins of Pragmatism: Studies in the Philosophy of Charles Sanders Peirce and William James" by A. J. Ayer.

simple elimination/erasure procedures. For example, "(a or a) and (~a)" clearly is a contradiction, as there are no conditions of truth for "a" under which the statement holds. In sentential calculus, this becomes immediately clear, by removing the repeated disjunct. Someone could use a truth table to verify this, but that would be overly pedantic. Similarly, in Peirce's system, one could continue the elaborate decision procedure outlined above to identify truth conditions, or one could simply follow his rules. Since Peirce's system takes disjunctions to be their corresponding conditionals, the initial statement is: "(~a then a) and ~a". In Peirce's system this becomes:



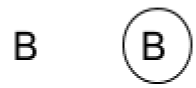
Then, by the Rule of Deiteration:



And then, by double negation elimination:

This is essentially identical to "a and ~a" in sentential calculus, and came about in a similarly similar procedure.

Interestingly, though, this case exhibits a way in which Peirce's system is still chained to the linguistic conventions he was trying to avoid. "a and ~a" is different from "b and ~b" just as 

is different from 

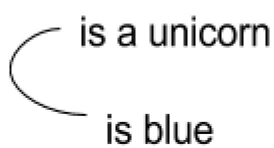
. It takes semantics to equate the two, in either case, since rules are not explicitly given to allow the replacement of one sentence letter with another. This is just the type of convention Peirce was attempting to avoid when creating a graphical system.

In the case of contradictions, the blot could possibly be a solution. The blot is a universal symbol for contradiction independent of sentence letters. Therefore, if we represented all contradictions within his system as blots, then his system would be freed of the linguistic conventions related to contradictions. But, while both of the above contradictions above could be represented using the blot, Peirce gives no rules allowing us to transform 

into a blot. If his system involved such a rule, then contradictions would be independent of language, as Peirce would like. In this way, Peirce seems to be missing the final step.

Imagine if the following Rule of Contradictions were added to Peirce's system: for any subgraph X, if both X and X surrounded by a cut appear on the SA, then the two subgraphs can be replaced by one blot. In this way, then, the ultimate depiction of contradictions would not involve sentence letters. Importantly, as well, this formulation of the rule does not involve an appeal to language. The rule could have been the following: for any sentence letter ‘a’, if both ‘a’ and ‘a’ surrounded by a cut appear on the SA, then replace both the letter and its negation with a blot. This rule would be intrinsically connected to the letters, and therefore would not be any different from a hypothetical parallel rule in the traditional system. My proposed Rule of Contradictions would maintain the importance of the visual in Peirce’s system, by considering full subgraphs and their negation, instead of simple sentence letters.

Peirce’s Existential Graphs satisfy many of his ultimate desires in constructing a diagrammatic method of logic. Beta graphs are built upon the above system with lines specifying individuals in the system, which can be seen as existential generalization. See the diagram below for an example of a beta graph representing “there is a blue unicorn”:



More literally this graphs represents “there is something that is a unicorn and that is blue”

As we can see, beta graphs remove the constraints of variable choice from normal logical notation, in favor of spatial diagrams.

Peirce’s diagrams embrace our natural understanding of reasoning. While reasoning flows in stages, each step need not be defined linearly. Peirce lays out proofs of logical statements in this way. Each step flows linearly from previous step using the Alpha Permissible Transformations, but within each step, the graphs can be written in any manner that preserves their “integrity.” Therefore, Peirce’s system involves formalization that faithfully reflects natural reasoning.

Peirce recognizes that his system is not bulletproof, however. At the start of his third lecture he admits, “there are probably better rules than that which I give; but it is the duty of my students to find these out” (Lecture 3 Draft 1). Peirce’s worry is well founded, since his system has required modification to avoid pitfalls. I will outline three major problems within Peirce’s Alpha Graphs, two of which could likely be remedied and one of which is intrinsic to the diagrammatic method. I will consider in turn objections to Peirce’s proof strategy, transformation rules, and diagrammatic methodology.

Let me begin with a consideration of Peirce’s proof strategy. Peirce outlines several proofs in his writing, both in formalized lecture notes and as exercises and solutions in his correspondences to Lady Welby, an uneducated Victorian aristocrat with whom Peirce corresponded frequently. In each proof steps are specifically separated from each other, with transformation rules given to justify the movement from one step to the next. As pointed out by Eric Hammer, this gives rise to a major problem in his proof strategy, as “This notion of provability does not allow one to refer back to the previous stages of a proof, say to use a version of conjunction introduction. Rather, one is restricted to making incremental changes to a single diagram” (Hammer 1996). Essentially, with each implementation of a transformation rule, one changes the original graph to a new graph. However, unlike normal deductive proofs, one is unable to refer back to or incorporate previous graphs into a future stage of the proof. For example one cannot assert two previous lines onto one new line, introducing a conjunction. In this way the Peircean proof strategy is “memoryless” (Norman), and consequently the only way to incorporate information gained during a step of the proof into a future step is to carry it “along as one proceeds” through every next step of the proof (Hammer 1996).

This “memoryless” quality would not be a drawback if the system were purely intended to facilitate reasoning. Essentially, memorylessness would not prevent us from reasoning easily, as much as from recording that reasoning easily. For example if we wanted to prove that something implies A&B by deriving A and B separately, it would not be problematic to derive A and then later derive B, and instead of including A&B in the final line, simply referring back to previous lines as having already been reasoned through. Here it is important to note Peirce’s intention in logical reasoning more generally. For Peirce, logic was not a tool to simplify reasoning. It was a method with which to formalize reasoning. Therefore, as this “memoryless” issue clearly limits the power of EG to record reasoning, it presents a problem to Peirce’s goals.

If this could not be remediated, it would clearly pose a fundamental problem to Peirce’s proof strategy, as this notion of provability seems disjoint from the more natural provability of other systems. It seems entirely natural to incorporate items learned in previous steps into future steps, but, despite desiring naturalness, Peirce’s system does not allow this. Of course, a solution has been presented. Jesse Norman proposed an altered proof strategy, which would consider the full proof to be a single graph (Norman). That way each stage of the proof could remain continuously relevant, by iterating the original graphs and then updating the iterations. If one iterates each stage, the information present in each stage can be preserved and then introduced later.[9] This builds in memory of previous lines, by removing the concept of lines entirely.

While this solution would help solve the "memoryless" problem, it acts counter to Peirce's ultimate goal in constructing a diagrammatic system. At the start of his second lecture, Peirce defines mathematical reasoning as the consideration of "elementary steps and how they are put together," where we "subdivide one step into as many as possible" (Lecture 2 Volume 1). Clearly Peirce values each step of reasoning separately. The goal of his diagrammatic system is to formalize each step of reasoning as naturally as possible. Peirce even explicitly champions the steps involved in his proof strategy, bragging that, "The fact that our system breaks this up into three steps goes to show that our main purpose, that of dissecting reasoning into its simplest elements, has been, in some measure, at least attained" (Lecture 2 Volume 2). Therefore this modified one graph approach, where the specification of steps is eliminated, would not be an instance of the mathematical reasoning to which Peirce strives. If one were to look at a proof already completed using Norman's strategy, it would simply look like a jumbled set of graphs on a page. No map of reasoning would be apparent whatsoever, as the stages would be blurred. While one could likely recreate the path of reasoning by analyzing pairs of graphs differing by one transformation, Peirce specifically desired his proofs to readily reflect the steps of reasoning. The one graph approach would specifically negate Peirce's articulated

[9] Clearly the word "later" is no longer relevant here, as proofs would not be linearly progressing in stages as much as simply being one large graph representing the full proof strategy. I use "later," however purely to simplify understanding.

goals. Consequently the unnatural "memoryless" quality of Peirce's graphs has not yet been solved in a way true to Peirce's own convictions.

Let us now step away from full proofs to consider the transformation rules themselves. As pointed out in Sun-Joo Shin's arguably uncharitable "The Iconic Logic of Peirce's Graphs," his rules are not entirely independent (Shin). The Rule of Iteration and Deiteration overlaps with the Rule of Insertion and Erasure in several different cases. By the rule of insertion, a graph is allowed to be inserted into an oddly enclosed area. Therefore, if a subgraph is oddly enclosed (enclosed by an odd number of cuts), by the rule of insertion it can be inserted again into that enclosed area, resulting in two graph-replicas of the subgraph within the oddly enclosed area. This is just the case of iterating the subgraph, allowed by the rule of iteration. Similarly, if there are two graph replicas of a subgraph on the SA, either evenly enclosed or not enclosed at all, by the rule of erasure one is allowed to erase one of the replicas. This could also be achieved by using the rule of deiteration, since the two graph replicas would be surrounded by the same number of cuts. In both of these frequently used cases, the transformation rules are not independent.

While this criticism does not negate the possible power of EG, it does show that the transformation rules are not as minimal as possible, a goal in most deductive systems. It reveals a lack of elegance and efficiency in Peirce's rules. Here it is important to note that the rules were self-admittedly imperfect. Peirce understood that "there are better rules out there" (Lecture 3 Draft 1). Compounding with this admission is the fact that Peirce's explanation of his rules resides primarily in lecture notes intended to be spoken, notes where Peirce confesses to sacrificing rigor for brevity due to the short lecture time. Therefore I have trouble seeing this as a fundamental flaw in the system, as much as a rut which may be remedied following Peirce's own intentions. I do question whether an independent set of rules could be presented that would preserve the naturalness of the graphs, however. If the rules were forced to be independent, they would likely become even more pedantic, making them even less intuitive and ultimately damaging the felt naturalness of the system.

Finally, let us move from the transformation rules to consider the signs themselves. As I laid out earlier, Peirce's diagrams are intended to remove the inherent and unnecessary restrictions imposed by traditional writing. I will argue, however, that diagrams impose their own unnecessary restrictions that ultimately negate their intended naturalness.

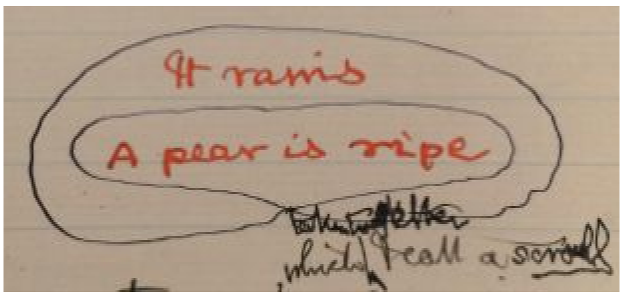
As the blackboard itself is a sign in the alpha system, Peirce is forced to consider empty graphs that look invisible. If a subgraph is scribed upon the SA, it is possible that that subgraph is surrounded by "empty" subgraphs (Lecture 2 Volume 2). He explicitly states that a blank graph can accompany any graph, a statement given as an aside, separate from all other rigorous accounts of the signs and transformation rules of the system. He exclaims, "it is truly fortunate that this is permitted, inasmuch as it would be physically impossible that a blank should not accompany every graph," showing that Peirce realized he needed to account for the invisible graph possibility (Lecture 2 Volume 2).

In traditional logical notation, there is plenty of blank space on the page next to the written strings. However, as there are no non-written signs, this blank space need not be accounted for. In Peirce's system the SA is logically important, introducing the importance of non-written, seemingly invisible signs. Therefore, as any graph could be surrounded by an array of invisible "blank graphs," Peirce must explicitly declare them to be permissible.

The addition of blank graphs to a graph does not have any logical import whatsoever. It would initially seem that the invisible blank graphs may act as tautologies, which can always be added. But the blot already represents tautologies. Peirce never specifies invisible, blank graphs as signs, and never gives them semantic meaning. Somehow, though, despite being invisible and logically unimportant, Peirce needed to account for these blank graphs rigorously. If the only signs in the system were scribed graphs, then there would not be an issue of blank graphs, as all signs would be explicitly visible. However, with the SA taken as a sign itself, invisible objects become important within the system. This seems to act counter to Peirce's ultimate goal of making the diagrammatic method as natural as possible. If one needs to account for invisible but ever present signs, the system no longer feels natural. Therefore this issue seems both fundamental to Peirce's diagrammatic methodology and counter to his desired naturalness.

The fact that his account of blank graphs is entirely disjoint from his notes on the formalization of the system signals that Peirce was not as formal in his account as it once seemed. Peirce explicitly laid out the signs and transformation rules of his system and the term “blank graph” was nowhere to be found. However he later rigorously accounts for blank graphs. It seems that Peirce was not as rigorous initially, forgetting or avoiding mentioning a subject that required formalization.

A similar inconsistency in formality can be seen in Peirce’s hand drawn graphs themselves. He begins by carefully drawing his scrolls, being sure to keep the two cuts meeting at the bottom.



(Lecture 2 Volume 1)

He steadfastly preserves this cut intersection throughout his second lecture notes, but his later diagrams look more like concentric ovals. Remember, though, that two graphs are equivalent if and only if they are topologically equivalent. This removal of an intersection clearly changes the scrolls topologically, damaging the topological “integrity” of the diagram. Despite their inequality, Peirce gives no rigorous rule for moving between them. Looking at the system semantically, it is clear to see that this intersection does not change the logical meaning of the scrolls once each cut represents an instance of negation. However, Peirce had explicitly defined graphs in terms of topological equivalence. Therefore this change in the layout of the scroll is one Peirce should have but never did account for.

In the case of traditional logical systems, one could write the logical signs in many different ways, in cursive or type for example. Importantly, though, topological equivalence plays no role in the notation of traditional logic. In Peirce's system, however, signs were defined by their topology. This forces Peirce to account for visual aspects of his system in ways unnecessary in other systems.

The rigor in EG is found in its visual, topological, layout. Therefore one must account rigorously for the visual aspects of the system, even those so extreme as invisible graphs and seemingly arbitrary notation changes.[10] While Peirce's graphs do avoid many of the problematic physical constraints of typical logical notation as desired, they add the same type of unimportant complexity he was attempting to elude. Formalized diagrams force a logician to account for topological constraints and invisible objects not whatsoever written down, both of which ultimately have little to no semantic importance. This makes the system feel unnatural, exactly going against Peirce's goal when creating EG. This problem is sadly one intrinsic to diagrammatic methods, and therefore, unlike the others, cannot be resolved by modifying rules. Diagrams are inherently visual, and therefore cannot escape visual constraints when defined in terms of their topology.

Peirce knew his graphs were imperfect, simply admitting, "I am a little disappointed with my graphs" (Manuscript 512). I have discussed three major areas that falter: "memoryless" proofs, redundant rules, and unnecessary physical constraints. I doubt, however, that Peirce's disappointment with his work resides in these errors. Remember that Peirce's primary goal was to free logic of its linguistic chains, to display logic in its more natural state without the constraints of our expression of it. While

[10] Peirce later goes so far as to declare a quality of EG as following "from physical necessity," an idea which seems to depart from his initial goal in diagrammatic notation of freeing logic of "physical" constraints (On the Simplest Branch of Mathematics, MA 2).

Peirce's graphs work toward this goal, they are necessarily still linguistically constrained. The Alpha Graphs successfully counteract the directionality constraints of classical notation. However, his use of sentence letters continues some of the linguistic constraints he was attempting to avoid, as was exhibited in the case of contradictions. While the proposed Rule of Contradiction counteracts this linguistic limitation in the case of contradictions, the inherent connection between language and logic remains. In this way, Peirce's Alpha Graphs act as a middle ground between traditional logical notation and a completely non-linguistic system, his ultimate goal. Peirce would need to present a system devoid of linguistic symbols entirely to achieve what he set out to do.

This system could not simply mirror the Alpha Graphs, but with different notation. For example, imagine a system identical to Peirce's, however, each assertion is represented by a different texture on the SA. A statement being asserted would be represented by the full SA being covered by this texture. The negation of that statement would be represented by that texture being constrained to an oval, like a cut in the original system. The conjunction of two statements would be the overlapping of textures, and the scroll-type shape would still represent the conditional. This system seems to be free of linguistic constraints, since it does not use any linguistic symbols. However, as this system simply mirrors Peirce's problematic alpha system, it still remains intrinsically connected to language. Sentences would still be transformed into graphical structures, and then those sentences would be analyzes for their logical properties. Therefore, despite language not being directly incorporated into the signs of the system, logical properties would still be defined in terms of relationships between linguistic sentences.

Peirce's Alpha Graphs helped bring logical structure to the forefront, by removing many of the constraints posed by normal, formal language notation. It displayed logical features substantially more directly than the traditional sentential calculus system. While the new system did have several structural flaws, of which I presented three, it more problematically preserved aspects of the connection between language and logic that Peirce had intended to remove. To avoid these pitfalls, a logical system would need to take logical properties first, and then apply language to them. Currently, sentences are analyzed to identify their logical properties. A system true to the fundamentality of logic would need to consider logical properties first, and then identify sentences which exhibit them. In this way logic could be taken as inherently foundational, a necessity of the world exemplified in language.

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